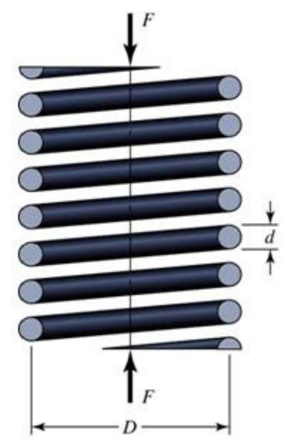
CH 10: Mechanical Springs

Springs are important mechanical elements because of their flexibility and controllable stiffness. Springs allow controlled application of force or torque; also they can be used for storing and releasing energy.

In general, springs may be classified as: wire springs, flat springs, and special shaped springs.

- Wire springs (round or square wires) are helical in shape and can be made to resist tension, compression, or torsion.

Stresses in Helical Springs

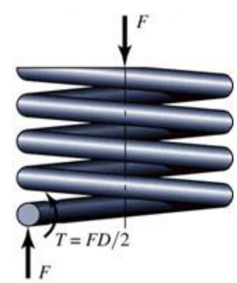


Consider a helical compression spring of mean coil diameter "D" and wire diameter "d" subjected to compressive force "F".

- If we remove a portion of the spring, the internal reactions will be a direct shear F and a torque $T = FD/2$ where each will cause a shear stress, and the maximum shear will occur at the inner surface of the wire which is equal to:

$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A}$$

Substituting $T = FD/2$, $r = d/2$, $J = \frac{\pi}{32} d^4$, $A = \frac{\pi}{4} d^2$ gives:



$$\tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

- Defining the spring index which is a measure of coil curvature as:

$$C = D/d$$

For most springs C ranges from 6 to 12

We get:

$$\tau = \frac{2C+1}{2C}\left(\frac{8FD}{\pi d^3}\right) = K_s \frac{8FD}{\pi d^3}$$

where $K_s = \frac{2C+1}{2C}$ is called the "*Shear stress correction factor*"

This equation assumes the spring wire to be straight and subjected to torsion and direct shear.

- However, the wire is curved and the curvature increases the shear stress and this is accounted for by another correction factor K_C and thus the equation becomes:

$$\tau = K_c K_s \frac{8FD}{\pi d^3}$$

where K_C is the "*curvature correction factor*".

- Or easier the two correction factors are combined together as a single correction factor $K_B$where:

$$\boxed{K_B = K_C K_S = \frac{4C+2}{4C-3}}$$

Thus;

$$\boxed{\tau = K_B \frac{8FD}{\pi\, d^3}}$$

Deflection of Helical Springs

The deflection-force relation can be obtained using *Castigliano's* theorem.

- The total strain energy in the spring wire has two components torsional and shear.

$$U = \frac{T^2 L}{2GJ} + \frac{F^2 L}{2AG}$$

Substituting for T, A & J and knowing that $L = \pi DN$

where $N = N_a$ is the *Number of active coils*, we get:

$$U = \frac{4F^2 D^3 N}{d^4 G} + \frac{2F^2 DN}{d^2 G}$$

- Applying *Castigliano's* theorem to get the deflection "y";

$$y = \frac{\partial U}{\partial F} = \frac{8FD^3 N}{d^4 G} + \frac{4FDN}{d^2 G}$$

since $C = D/d$ we can write:

$$y = \frac{8FD^3N}{d^4G}\left(1+\overbrace{\frac{1}{2C^2}}^{\textit{very small}}\right) \approx \frac{8FD^3N}{d^4G}$$

The effect of transverse shear is neglected

- Knowing that the "<u>spring rate</u>" $k = F/y$

  Thus,

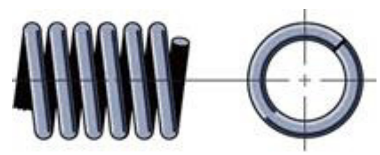
$$\boxed{k = \frac{d^4G}{8D^3N_a}}$$

N_a: *Number of active coils*

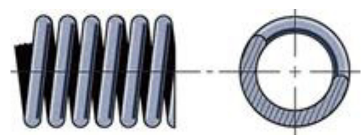
<u>Compression Springs</u>

There are four types of ends used for compression springs:

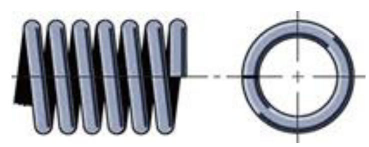
- *<u>Plain ends</u>*: ends are non-interrupted (same as if the spring was cut into sections).



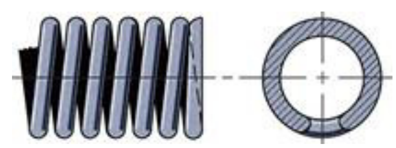
- *<u>Plain-Ground ends</u>*: plain ends that are grinded flat.



- *<u>Squared (or closed) ends</u>*: ends are squared by deforming them to zero degree helix angle.



- *<u>Squared and Ground ends</u>*: ends are grinded after squaring.



❖ *<u>Table 10-1</u>* gives the <u>dimension formulas</u> (*free length, solid length, pitch*) and the <u>number of active coils</u> N_a for the different types of ends.

The max compression of spring = free length (l_o) - solid length (l_s)

<u>Stability of compression springs</u>:

Similar to columns, compression springs <u>may buckle</u> if the deflection (*i.e., load*) becomes too large.

- The critical value of deflection (i.e., *the value causing buckling*) is given as:

$$y_{cr} = l_0 C_1' \left[1 - \left(1 - \frac{C_2'}{\lambda_{eff}^2} \right)^{1\backslash 2} \right]$$

where C_1' and C_2' are elastic constants;

$$C_1' = \frac{E}{2(E-G)} \quad \text{and} \quad C_2' = \frac{2\pi^2(E-G)}{2G+E}$$

and λ_{eff} is the *effective slenderness ratio*;

$$\lambda_{eff} = \alpha l_0 \backslash D$$

where "α" is the end condition constant (*see Table 10-2*)

- Absolute stability is obtained when $(C_2' \backslash \lambda^2{}_{eff}) > 1$ (i.e., y_{cr} gives complex number)

$$\Rightarrow C_2' > \lambda_{eff}^2$$

Thus,

$$l_0 < \frac{\pi D}{\alpha} \sqrt{\frac{2(E-G)}{2G+E}}$$

Buckling will not occur if this condition is satisfied

- For steels, this turns out to be: $l_0 < 2.63 \dfrac{D}{\alpha}$
  - If ends are ground and squared (*α=0.5*) it becomes: $l_0 < 5.26D$

Spring Materials

Springs are manufactured using hot (or cold) working processes depending on size of the wire and spring index.

- A variety of materials may be used for marking springs, *Table 10-3* gives description of the most commonly used steels.

- Spring materials may be compared based on their tensile strength. However, the tensile strength for wires depends on the wire diameter, and the strength-diameter relation is:

"Ultimate" tensile strength ⟶ $S_{ut} = A / d^m$

A & m are material constants

- ❖ *Table 10 - 4* gives the material constants for different wire materials.
- ❖ The diameters for standard gage wires are found in *Table A-28*.

- However, springs are subjected to shear not tension and we need to consider yield strength not ultimate strength.

  An approximate relation between ***shear yield strength*** S_{ys} and *ultimate tensile strength* S_{ut} is: $0.35 S_{ut} \le S_{ys} \le 0.52 S_{ut}$

- ❖ Table 10-6 gives a better approximation of the relation between S_{ys} and S_{ut} for different materials.
- ❖ Table 10-5 gives the elastic constants E & G for different spring materials.

*See **Example 10-1** from text*

Design of Helical Compression Springs for Static Service

Make "a priori" decisions (*if no specific requirements are given*):

- ***Material:*** *HD* steel should be the first choice since it has the lowest relative cost.
- ***Function:*** maximum load and spring stiffness or maximum displacement.
- ***Type of ends:*** squared ends should be the first choice since it gives good stability and has low cost.
- ***Manufacturing:*** as-wound should be the first choice since it has the lowest cost.
- ***Safety:*** use a design factor at solid length of at least 1.2

$$(n_s)_d \ge 1.2$$

- ***Working range:*** to ensure linearity we should avoid closing the spring to its solid length under the maximum load. *why?*

  It is recommended to confine the operating range of the spring to the central 75% of its possible compression distance (i.e., between $F = 0$ and $F = F_s$)

Thus we can write:

$$F_s = (1+\xi)F_{\max}$$

where F_s is the force needed to compress the spring to solid length, and ξ is the fractional overrun to closure.

- It is recommended that $\xi \geq 0.15$

➢ Thus, the free length of the spring is found as: $l_o = l_s + (1+\xi)y_{max}$

- If the spring runs <u>over-a-rod</u> *or* <u>in-a-hole</u>, we can we use their diameters as governing value for the coil *ID* or *OD* (*with some tolerance included*).
- However, in cases where there is <u>no constraints</u> on coil diameter, we can solve for it by setting the shear yield strength (with design factor considered) to be equal to the maximum shear stress at solid length.

Only for As-wound:

$$\frac{S_{ys}}{(n_s)_d} = K_B \frac{8F_s D}{\pi d^3}$$

substituting and solving for *C* gives:

$$C = \frac{2\alpha - \beta}{4\beta} + \sqrt{\left(\frac{2\alpha - \beta}{4\beta}\right)^2 - \frac{3\alpha}{4\beta}}$$

where , $\alpha = \dfrac{S_{ys}}{(n_s)_d}$ and $\beta = \dfrac{8(1+\xi)F_{\max}}{\pi d^2}$

and thus: $D = Cd$

- The recommended ranges for <u>spring index</u> and number of <u>active turns</u> are:

$$4 \leq C \leq 12 \quad \& \quad 3 \leq N_a \leq 15$$

- In addition, a "*<u>figure of merit</u>*" that depends on the material cost and weight can be used to select between the different feasible designs.

$$fom = -(relative\_material\_\cos t)\frac{\pi^2 d^2 N_t D}{4}\gamma$$

"fom": the <u>closer to zero</u> the better

where γ is the material weight density

Design Strategy:

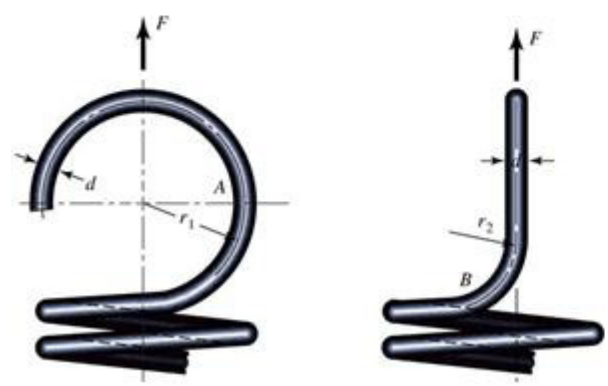
- Consider all constraints and make a priori decisions.
- Make the wire diameter as your design variable and choose initial wire diameter.
- Compute the other parameters: *D*, *C*, *OD* or *ID*, N_a, l_s, l_o, $(l_o)_{cr}$, n_s, *fom.*
- Choose other wire diameters and recalculate the other parameters and tabulate the data.
- Use the given constraints and recommended ranges of the parameters to identify the feasible designs, then use the "*fom*" to choose between the feasible designs.
  - See the design flow chart (*fig. 10-3*) in text.

See ***Example 10-2*** *from text*

Extension Springs

Extension springs are used to carry tensile loading; they require some means to transfer the tensile load to the body of the spring, such as threaded plug or swivel hook (*see fig. 10-6*)

- Extension springs are made such that the body coils are touching each other and the spring usually has pre-tension.
- Stresses in the body are handled the same as compression springs.
- Also, stresses in the hook need to be considered.



  - Maximum tensile stress will occur at *point "A"* on the inner surface of the ring:

Bending stress Axial stress

$$\sigma_A = (K)_A \frac{16FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

Where $(K)_A$ is the bending stress correction factor for curvature.

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} \quad \& \quad C_1 = \frac{2r_1}{d}$$

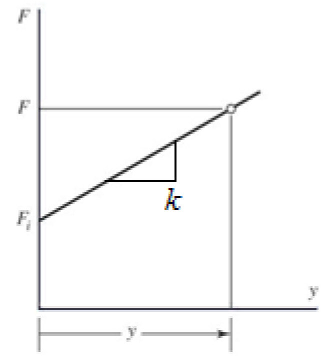
Usually $r_1 = D \backslash 2$
D: mean coil diameter

  - Maximum shear stress will occur at *point "B"*

$$\tau_B = (K)_B \frac{8FD}{\pi d^3}$$

Where $(K)_B$ is correction factor for curvature and direct shear.

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} \qquad \& \qquad C_2 = \frac{2r_2}{d}$$



- Extension springs usually have "*initial tension*" F_i, and thus the load-deflection relation is:

$$F = F_i + ky$$

  - Due to the initial tension in the spring, there is torsional shear stress always present in the coil.
    - The preferred "*range*" of the "*uncorrected*" initial shear stress is:

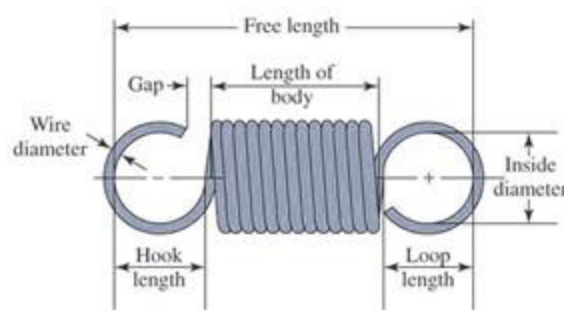
$$\tau_i = \frac{231}{e^{0.105C}} \pm 6.9\left(4 - \frac{C-3}{6.5}\right)$$ *MPa* where C is the spring index

- When determining the stiffness of the spring "k", the deformation of the hooks needs to be accounted for and thus an *"equivalent" active number of turns* is used:

$$N_a = N_b + \frac{G}{E}$$ where N_b is the number of body coils

- The free length of an extension spring with ordinary twisted end loops is found as:

$$l_0 = 2(D-d) + (N_b + 1)d = (2C - 1 + N_b)d$$



❖ *Table 10-7* gives the maximum allowable shear stress for extension springs (*as percentage of the tensile strength*).

*See **Example 10-6** from text*